

# Casimir scaling as a test of QCD vacuum.

V.I. Shevchenko, Yu.A. Simonov

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## Abstract

Recent accurate measurements [1] of static potentials between sources in various representations of the gauge group  $SU(3)$  provide a crucial test of the QCD vacuum models and different approaches to confinement. The Casimir scaling of the potential observed for all measured distances implies strong suppression of higher cumulant contributions. The consequences for the instanton vacuum model and the spectrum of the QCD string are also discussed.

## 1 Introduction

The structure of QCD in its nonperturbative domain commands attention of theorists for many years. Confinement and chiral symmetry breaking have been studied both using theoretical models and lattice simulations (for review see [2]).

However most of the models are designed to describe confinement of colour charge and anticharge in the fundamental representation of the gauge group  $SU(3)$ , i.e. the area law for the simplest Wilson loop and hence linear potential between static quark and antiquark. A supplementary and as we shall see below, very important information about QCD vacuum is provided by the investigation of interaction between static charges in higher  $SU(3)$  representations. Comparing static potentials for different charges one can derive information about field correlators in the vacuum, which is not possible to obtain from fundamental charges alone.

The recent accurate measurements of the corresponding potential have been performed by G.Bali in [1]. Preliminary physical analysis of the data

from [1] was presented in [3]. We present in this paper more extended investigation and discuss new important information about the QCD vacuum and constraints on several QCD vacuum models.

## 2 Casimir scaling of the static potential

We define static potential between sources at the distance  $R$  in the given representation  $D$  as:

$$V_D(R) = - \lim_{T \rightarrow \infty} \frac{1}{T} \ln \langle W(C) \rangle, \quad (1)$$

where the Wilson loop  $W(C)$  for the rectangular contour  $C = R \times T$  in the "34" plane admits the following expansion [4]

$$\begin{aligned} \langle W(C) \rangle &= \left\langle \text{Tr}_D \text{P exp} \left( ig \int_C A_\mu^a T^a dz_\mu \right) \right\rangle = \\ &= \text{Tr}_D \exp \sum_{n=2}^{\infty} \int_S (ig)^n \langle \langle F(1)F(2)..F(n) \rangle \rangle d\sigma(1)...d\sigma(n) \end{aligned} \quad (2)$$

Nonabelian Stokes theorem [5] has been used in the above expression with the notation  $F(k)d\sigma(k) = \Phi(x_0, u^{(k)})E_3^a(u^{(k)})T^a\Phi(u^{(k)}, x_0)d\sigma_{34}(u^{(k)})$ , where  $\Phi$  is a parallel transporter and  $x_0$  is an arbitrary point on the surface  $S$  bound by the contour  $C$ . The double brackets  $\langle \langle ... \rangle \rangle$  denote irreducible Green's functions proportional to the unit matrix in the colour space.<sup>1</sup>

Since (2) is gauge-invariant, it is convenient to make use of generalized contour gauge [6], which is defined by the condition  $\Phi(x_0, u^{(k)}) \equiv 1$ .

The  $SU(3)$  representations  $D = 3, 8, 6, 15a, 10, 27, 24, 15s$  are characterized by  $3^2 - 1 = 8$  hermitian generators  $T^a$  which satisfy the commutation relations  $[T^a, T^b] = if^{abc}T^c$ . One of the main characteristics of the representation is an eigenvalue of quadratic Casimir operator  $\mathcal{C}_D^{(2)}$ , which is defined according to  $\mathcal{C}_D^{(2)} = T^a T^a = C_D \cdot \hat{1}$ . Following the notations from [1] we introduce the Casimir ratio  $d_D = C_D/C_F$ , where the fundamental Casimir

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<sup>1</sup>It makes unnecessary to write colour ordering operator in the r.h.s. of (2). Notice also, that the averages in (2) refer to a single Wilson loop and do not take into account screening effects which add to  $W(C)$  multiloop contributions, as will be explained below in section 4.

$C_F = (N_c^2 - 1)/2N_c$  equals to 4/3 for  $SU(3)$ . The invariant trace is given by  $\text{Tr}_D \hat{1} = 1$ .

Since a simple algebra of the rank  $k$  has exactly  $k$  primitive Casimir–Racah operators [7] of order  $m_1, \dots, m_k$ , it is possible to express those of higher order in terms of the primitive ones. In the case of  $SU(3)$  the primitive Casimir operators are given by

$$\mathcal{C}_D^{(2)} = \delta_{ab} T^a T^b \ ; \ \mathcal{C}_D^{(3)} = d_{abc} T^a T^b T^c \quad (3)$$

while the higher rank Casimir operators are defined as follows

$$\mathcal{C}_D^{(r)} = d_{(i_1 \dots i_r)}^{(r)} T^{i_1} \dots T^{i_r} \quad (4)$$

where the totally symmetric tensor  $d_{(i_1 \dots i_r)}^{(r)}$  on the  $SU(N_c)$  is expressed in terms of  $\delta_{ik}$  and  $d_{ijk}$  (see, for example, [8]).

The potential (1) with the definition (2) admits the following decomposition

$$V_D(R) = d_D V^{(2)}(R) + d_D^2 V^{(4)}(R) + \dots, \quad (5)$$

where the part denoted by dots contains terms, proportional to the higher powers of the quadratic Casimir as well as to higher Casimirs.

The fundamental static potential contains perturbative Coulomb part, confining linear and constant terms

$$V_D(R) = \sigma_D R - v_D - \frac{e_D}{R} \quad (6)$$

The Coulomb part is now known up to two loops [9] and is proportional to  $C_D$ . The "Casimir scaling hypothesis" [10] declares, that the confinement potential is also proportional to the first power of the quadratic Casimir  $C_D$ , i.e. all terms in the r.h.s. of (5) are much smaller than the first one. In particular, for the string tensions one should get  $\sigma_D/\sigma_F = d_D$ .

This scaling law is in perfect agreement with the results found in [1]. Earlier lattice calculations of static potential between sources in higher representations [10] are in general agreement with [1].

To see, why this result (to be more precise – why the impressive *accuracy* of the "Casimir scaling" behaviour) is nontrivial, let us examine the colour structure of a few lowest averages in the expansion (2).

The first nontrivial Gaussian cumulant in (2) is expressed through  $C_D$  and representation-independent averages as

$$\text{Tr}_D \langle F(1) F(2) \rangle = \frac{C_D}{N_c^2 - 1} \langle F^a(1) F^a(2) \rangle = \frac{d_D}{2N_c} \langle F^a(1) F^a(2) \rangle, \quad (7)$$

so Gaussian approximation satisfies "Casimir scaling law" exactly. It is worth being mentioned, that this fact does not depend on the actual profile of the potential. It could happen, that the linear potential observed in [1] is just some kind of intermediate distance characteristics and changes the profile at larger  $R$  (as it actually should happen in the quenched case for the representation of zero triality due to the screening of the static sources by dynamical gluons from the vacuum, or, in other words, due to gluelumps formation). The coordinate dependence of the potential is not directly related to the Casimir scaling, and can be analyzed at the distances which are small enough to be affected by the screening effects.<sup>2</sup>

Having made these general statements, let us come back to our analysis of the contributions to the potential from different field correlators. We turn to the quartic correlator and write below several possible colour structures for it. We introduce the following abbreviation

$$\langle F^{[4]} \rangle = \langle F^a(1)T^a F^b(2)T^b F^c(3)T^c F^d(4)T^d \rangle = T^a T^b T^c T^d \langle F^{[4]} \rangle^{abcd}$$

where the Lorentz indices and coordinate dependence are omitted for simplicity of notation. One then gets, with some work in the last case the following possible structures

$$\begin{aligned} \langle F^{[4]} \rangle^{abcd} &\sim \delta_{ab} \delta_{cd} & \langle F^{[4]} \rangle &\sim C_D^2 \cdot \hat{1} \\ \langle F^{[4]} \rangle^{abcd} &\sim \delta_{ac} \delta_{bd} & \langle F^{[4]} \rangle &\sim \left( C_D^2 - \frac{1}{2} N_c C_D \right) \cdot \hat{1} \\ \langle F^{[4]} \rangle^{abcd} &\sim f_{ade} f_{cbe} & \langle F^{[4]} \rangle &\sim -\frac{1}{4} N_c^2 C_D \cdot \hat{1} \\ \langle F^{[4]} \rangle^{abcd} &\sim f_{ace} f_{bde} & \langle F^{[4]} \rangle &= 0 \\ \langle F^{[4]} \rangle^{abcd} &\sim f_{apm} f_{bpn} f_{dem} f_{cen} & \langle F^{[4]} \rangle &\sim \frac{9}{4} \left( C_D^2 + \frac{1}{2} C_D \right) \cdot \hat{1} \end{aligned}$$

The operator  $\mathcal{C}_D^{(3)}$  enters together with  $\mathcal{C}_D^{(2)}$  at higher orders. Notice, that the terms, proportional to the square of  $C_D$  appear in both the  $\delta\delta$  parts (the first and the second strings) and higher order interaction parts (the last string). Mnemonically the  $C_D$  – proportional components arise from the diagrams where the noncompensated colour flows inside while the  $C_D^2$  – components describe the interaction of two white objects. It is seen, that the Casimir scaling does not mean "quasifree gluons", instead it means roughly speaking "quasifree white multipoles" (see discussion at the end of the paper).

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<sup>2</sup>The same is true for the general criticism [11] of the confining potential evaluation on the lattice. The considered potential might be even different from linear but still demonstrate Casimir scaling.

Table 1: The Casimir–scaling and Casimir–violating string tensions and shifts. Based on the data from G.Bali, hep-lat/9908021.

$D$	$\sigma_D^{(4)} \cdot 10^4$	$\Delta\sigma_D^{(4)} \cdot 10^4$	$v_D^{(4)} \cdot 10^4$	$\Delta v_D^{(4)} \cdot 10^4$	$ \sigma_D^{(4)}/\sigma_D^{(2)} $	$\chi^2/N$
8	-3.486	1.2	-2.513	2.8	0.004	19.22 / 43
6	-6.428	1.2	0.950	2.6	0.007	25.76 / 42
15a	-5.244	0.55	-0.5611	1.1	0.003	39.06 / 42
10	-4.925	0.50	0.2489	1.0	0.003	22.05 / 41

Let us analyse the data from [1] quantitatively. We have already mentioned, that the Coulomb potential between static sources is proportional to  $C_D$  up to the second loop (and possibly to all orders, this point calls for further study) and hence we expect contributions proportional to  $C_D^2 \sim d_D^2$  to the constant and linear terms, i.e. we rewrite (7) as follows

$$V_D(R) = d_D V^{(2)}(R) + d_D^2 (v_D^{(4)} + \sigma_D^{(4)} R). \quad (8)$$

and all higher contributions are omitted. Here  $v_D^{(4)}, \sigma_D^{(4)}$  measure the  $d_D^2$ –contribution of the cumulants higher than Gaussian to the constant term and string tension respectively. The results of the fitting of the data from [1] with (8) for some representations are shown in the Table 1. See also Fig.1, where the quantity  $(V_D(R) - d_D V_F(R))$  versus distance  $R$  is depicted. This figure shows the same data as the Figure 2 from the paper [1].

All numbers in the table 1 are dimensionless and given in lattice units. The author of [1] used anisotropic lattice with the spatial unit  $a_s^{-1} = 2.4 GeV$ .

Several comments are in order. First of all it is seen that the Casimir scaling behaviour holds with very good accuracy, better than 1% in all cases

in the table 1 with the reasonable  $\chi^2$ . It should be stressed, that any possible systematic errors which could be present in the procedure used in [1] must either obey the Casimir scaling too or be very tiny, otherwise it would be unnatural to have the matching with such high precision.<sup>3</sup> Nevertheless, the terms violating the scaling are also clearly seen. While the value of the constant term  $v_D^{(4)}$  is found to be compatible with zero within the error bars, it is not the case for  $\sigma_D^{(4)}$ . We have not found any sharp dependence of  $\sigma_D^{(4)}$  on the representation  $D$ , which confirms the validity of the expansion (8) and shows, that the omitted higher terms do not have significant effect in this case. Notice the negative sign of the string tension correction. In euclidean metric it trivially follows from the fact, that the fourth order contribution is proportional to  $(ig)^4 > 0$  while the Gaussian term is multiplied by  $(ig)^2 < 0$  for real  $g$ .

### 3 Casimir scaling and instantons

From perturbation theory it follows, as it was already mentioned that Casimir scaling holds up to the  $g^6$  terms. One might suspect therefore that also non-perturbative configurations when treated exactly, ensure the Casimir scaling. This is not true however for the models based on the classical solutions. As an example of the model which violates scaling we mention here the model of the dilute instanton gas.

In the simplest  $SU(2)$  case the field strength of one instanton in the regular gauge is given by

$$gF_{\mu\nu}^a(x, z) = -\frac{\eta_{a\mu\nu} 4\pi\rho^2}{[(x - z)^2 + \rho^2]^2} \quad (9)$$

where  $z_\mu$  is the position of the instanton and  $\rho$  is its size, choosing the Wilson plane to be "12" one has  $\eta_{a12} = \delta_{a3}$ . The average over stochastic ensemble of the dilute instanton gas implies averaging over (global) color rotation of each instanton  $F_{12} \rightarrow \Omega^\dagger F_{12} \Omega$ , averaging over instanton positions  $z_\mu^{(k)}$   $k = 1..N$ , where  $N$  is the total number of instantons and antiinstantons in the volume  $V$  and weighted averaging over instanton sizes  $\rho$ . The latter one is assumed to be performed as the last step of all calculations. To the lowest order in density

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<sup>3</sup>Since we are mostly interested in the relative quantities, their actual magnitude in the physical units is of no prime importance for us. This is another reason why we do not discuss possible systematical errors of [1] and finite volume effects.

$\left(\frac{N}{V}\rho^4\right)$  one must consider one instanton and sum up over  $k, 1 \leq k \leq N$ , hence one can write [12]

$$\langle F(1)..F(2) \rangle = \langle T_{\alpha\beta}^3..T_{\rho\omega}^3 \rangle_{\Omega} \cdot \frac{N}{V} \int d^4z F^3(x_1, z)..F^3(x_n, z) \quad (10)$$

The lowest order terms read

$$\langle T_{\alpha\beta}^3 T_{\beta\gamma}^3 \rangle_{\Omega} = \delta_{\alpha\gamma} \frac{C_D}{3} \quad ; \quad \langle T_{\alpha\beta}^3 T_{\beta\gamma}^3 T_{\gamma\delta}^3 T_{\delta\epsilon}^3 \rangle_{\Omega} = \delta_{\alpha\epsilon} \frac{3C_D^2 - C_D}{15}$$

Consider now the rectangular Wilson loop  $R \times T$  and perform the expansion of the Wilson loop with respect to the  $\Omega$ - and  $z_{\mu}$  - average procedures:

$$\langle W \rangle_{\Omega, z_{\mu}} = \text{Tr}_D \left\langle \text{P exp } ig \int_S F_{12} d\sigma_{12} \right\rangle_{\Omega, z_{\mu}} = \text{Tr}_D \exp(-\Lambda_2 + \Lambda_4 + ..) \quad (11)$$

The terms  $\Lambda_2$  and  $\Lambda_4$  generate the following terms in the potential:

$$V_D(R) = V_D^{(2)}(R) + V_D^{(4)}(R) \quad (12)$$

where  $V_D^{(2)}$  is proportional to  $C_D$  and behaves as  $R^2$  at small  $R$ , namely  $V_D^{(2)}(R) = \bar{\gamma}^{(2)} R^2 / \rho^3$  where  $\bar{\gamma}^{(2)} = \frac{\pi}{32} \gamma^{(2)}$  with  $\gamma^{(2)} = \frac{C_D}{3} 16\pi^3 \frac{N}{V} \rho^4$ . For large distances  $R \gg \rho$  one has  $V_D^{(2)}(R) = \sigma^{(2)} R$  where  $\sigma^{(2)} = \gamma^{(2)} / 2\rho^2$ .

Analogously for  $V_D^{(4)}(R)$  one has

$$V_D^{(4)}(R) = -\frac{1}{24} \lim_{T \rightarrow \infty} \int d^4z \frac{(4\rho^2)^4 N}{T} \left( \frac{3C_D^2 - C_D}{15} \right) J(R, T)^4 \quad (13)$$

where

$$J(R, T) = \int_0^T \int_0^R \frac{dx dt}{((x - z_1)^2 + (t - z_4)^2 + z_{\perp}^2 + \rho^2)^2} \quad (14)$$

Straightforward calculation gives at large distances

$$V_D^{(4)}(R) = \sigma_D^{(4)} R \quad , \quad \sigma_D^{(4)} = \frac{N}{V} \rho^2 \frac{16\pi^4}{9} \frac{3C_D^2 - C_D}{15} \quad (15)$$

while in the regime  $R \ll \rho, T \gg \rho$  one gets

$$V_D^{(4)} = -\frac{N}{V} \frac{R^4}{\rho} \frac{\pi^6}{320} \frac{3C_D^2 - C_D}{15} \quad (16)$$

It is clear from (15) and (16), that linear asymptotics of  $V_D^{(4)}$  at large  $R$  occurs rather late, for  $R \geq 7\rho$ . One concludes, that the Casimir scaling of the potential is violated by the term  $V_D^{(4)}$ . This was interpreted in [3] as the upper bound on the instanton density, for  $v_D^{(4)} \approx 10^{-3} \text{Gev}$  it gives  $N/V \approx 0.2 \text{ fm}^{-4}$ , which is much less than the instanton density typically used in the literature  $N/V \sim 1 \text{ fm}^{-4}$ .

The original  $SU(3)$  case for the quark–antiquark potential in the dilute instanton gas approximation was considered in [13]. One has the following expression for the potential between static sources:

$$V(R) = 4\pi \frac{N}{V} \int_0^\infty d\rho \nu(\rho) \rho^3 \frac{1}{d(D)} \sum_{J \in D} (2J+1) F_J(x) \quad , \quad x = \frac{R}{2\rho} \quad (17)$$

and the function  $F_J(x)$  is given by some cumbersome double integral which can be found in [13]. Here  $d(D) \equiv D$  is the dimension of the representation  $D$  and sum over  $J = 0, \frac{1}{2}, 1, \dots$  goes over all  $SU(2)$  multiplets for decomposition of the given  $SU(3)$  representation with the corresponding weights. One has  $\sum_{J \in D} (2J+1) = d(D)$  and also

$$d(D) \cdot C_D = \frac{N_c^2 - 1}{3} \sum_{J \in D} J(J+1)(2J+1)$$

At small  $x$  the functions  $F_J(x) \sim x^2$ , while at large  $x$  the functions  $F_J(x)$  tend to  $J$ -dependent constant [13].

Numerically one finds at small distances

$$V(R) = 1.79 \cdot \gamma R^2 \cdot \epsilon_D + \mathcal{O}(R^4) \quad (18)$$

where  $\gamma = \pi \frac{N}{V} \int_0^\infty d\rho \nu(\rho) \rho$  and numerical coefficients  $\epsilon_D$  for  $D = 3, 8, 10$  are given by

$$\epsilon_3 : \epsilon_8 : \epsilon_{10} = 1 : 1.87 : 3.11$$

instead of Casimir scaling results  $1 : 2.25 : 4.5$ .

Similar situation takes place for the large distance asymptotics of the instanton–induced potential. It violates Casimir scaling on the level of 20% (see [13]) and can be excluded by the present analysis at the level of  $10\sigma$ .

So one can see the sharp contradiction between the dilute instanton gas model calculation for the quark–antiquark potential and the Casimir scaling of this potential found on lattice. This can be understood in one of two ways.



Either instantons are strongly suppressed in the real(hot) QCD vacuum (as it was observed in [14]) while they are recovered by the cooling procedure. Or else instanton medium is dense and strongly differs from dilute instanton gas, in such a way that higher cumulant components of such collectivized instantons are suppressed. Interesting to note, that linear confinement missing in the dilute gas, is recovered in this case.

## 4 Casimir scaling and QCD string

There is another important consequence of the observed Casimir scaling. It comes from the analysis of the confinement potential as being induced by the QCD string. In this case one has additional contribution to the confining potential besides the leading linear term, which comes from the internal dynamics of the string, in particular, from the transverse worldsheet vibrations. The simplest model in this respect is the Nambu–Goto string which action is proportional to the area of the surface bounded by the static sources worldlines. It modifies the confining potential with respect to the classical case (nonvibrating string) as

$$\sigma R \rightarrow \sigma R - \frac{\pi}{12} \frac{1}{R} + \dots \quad (19)$$

where the term  $-\pi/(12 \cdot R)$  will be referred to as the String Vibration (SV) term [15]. Despite the Nambu–Goto string model cannot be rigorously defined in  $D = 4$ , and, in particular the expansion of the r.h.s. of (19) meets singularity at the distances  $R \sim 1/\sqrt{\sigma}$  it is instructive to look whether or not the data [1] support the existence of such term. It is also worth noting, that the dimensionless coefficient  $-\pi/12$  is determined by the only two factors: target space dimension and the chosen string model. Having both factors fixed, it cannot be freely adjusted. Assuming  $\sigma_D = d_D \sigma_F$ , it is easy to see, that the Nambu–Goto induced SV term violates Casimir scaling.

It is a nontrivial task to separate the contributions of the discussed sort in the confining potential as it is because these corrections are essentially large distance effect, where they are subleading. But they have to become pronouncing in the expression (8) due to scaling violation. Namely one has

$$V_D(R) - d_D V_F(R) = (d_D - 1) \frac{\pi}{12} \frac{1}{R} + \dots \quad (20)$$

where the dots denote the terms, omitted in (19). The dashed line on Fig.1 corresponds to the r.h.s. of (20). It is seen, that Casimir scaling-violating SV term of the form (19) is completely excluded for all measured distances.<sup>4</sup>

We should mention at this point, that the lattice measurements performed in [16] did also demonstrate no fingerprints of the SV corrections of the form discussed.

One can see different explanations of this result. First of all, nobody has proved up to now, that the simplest bosonic Nambu–Goto string model properly describes the dynamics of the QCD string and theoretical background of (19) is not clear. Just the opposite is true – there are many reasons, why it is not the case (see discussion in [17]). The theory of the QCD string – whatever it will be – must explain the observed scaling of the potential.

## 5 Screening and string breaking

In the previous discussion the effect of string breaking in the triality zero representation was not taken into account. The modification of the potential (6) due to this effect was considered in [18] in the strong coupling expansion, resulting in the expression for the adjoint Wilson loop in the large  $N_c$  approximation

$$\langle W_8(C) \rangle = \exp(-\sigma_8 \cdot \text{area}(C)) + \frac{1}{N_c^2} \exp(-4\sigma_3 \cdot \text{perimeter}(C)) \quad (21)$$

The appearance of the second term on the r.h.s. of (21) signals screening due to the lattice diagrams related to the vacuum average of the product of two fundamental Wilson loops, while the first term is represented by the only one loop and has the cumulant expansion (2). We now demonstrate that a similar form also appears in the general background perturbation theory [22], and estimate the corresponding gluelump masses. To this end we represent  $A_\mu = B_\mu + a_\mu$  and use the 't Hooft identity to average separately over  $B_\mu$  and  $a_\mu$ , while gluon action is written as

$$S(B, a) = S_0(B) + S_1 + S_2 + S_{int} \quad (22)$$

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<sup>4</sup> Notice, that even the sign of the leading Nambu–Goto string correction is opposite to what has actually been observed.

In this expression  $S_1$  is proportional to the first power of  $a_\mu$ , while for  $S_2$  one has

$$S_2 = a^\mu \left( G[B]^{-1} \right)_{\mu\nu} a^\nu \quad (23)$$

and  $S_{int}$  contains higher order terms in the field  $a_\mu$ . We integrate over  $\mathcal{D}a_\mu$ , making perturbative expansion in  $S_{int}$ , which leads to

$$\langle W(B + a) \rangle = \int \mathcal{D}B \exp(-S(B)) [\text{Det } G[B]]^{-\frac{1}{2}} (W(B) + \dots) \quad (24)$$

where dots stand for higher order terms in the expansion over perturbative fields. Note that the retained in (24) term is purely nonperturbative. Using the world-line Fock–Schwinger representation for  $\text{Det } G$ , one has

$$\begin{aligned} \text{Det } G = \exp(\text{Tr } \ln G) = \exp \left( \text{Tr} \int_0^\infty \frac{ds}{s} \int \mathcal{D}z_\mu \exp(-K) \cdot \right. \\ \left. \cdot W_{C_z}(B) \exp \left( \int_C d\tau \, 2g\hat{F} \right) \right) \end{aligned} \quad (25)$$

where the integral is to be taken with the standard Wiener measure. It is seen from (25) and (24) that expansion of the exponent yields expansion in products of Wilson loops, and the first terms are

$$\langle W_C(B + a) \rangle = \langle W_C(B) \rangle + \langle \langle W_C(B) W_{C_z}(B) \rangle \rangle + \dots \quad (26)$$

where the average in the last term include the average over  $C_z$  according to (25). The second term in (26) is responsible for the screening, since the asymptotics of it for large  $C$  is of perimeter type, rather than area law. To find the logarithm of (26) explicitly, one can use the Hamiltonian formalism, obtained via the transition

$$\int \mathcal{D}z \exp \left( - \int d^4x \, L \right) = \langle x | \exp(-HT) | y \rangle \quad (27)$$

Then for the asymptotics of the second term in (26) one gets

$$\langle \langle WW \rangle \rangle = \text{const} \cdot \exp(-2M_{GL} \cdot T) \quad (28)$$

For simplified estimates we disregard the last spin term in (25) and interaction of two adjoint gluelumps with masses  $M_{GL}$ . To find  $M_{GL}$  from  $H$  one

can use the standard technic of einbein formalism (see [22] and references therein) to get an estimate [18]

$$M_{GL} \approx 1.4 \text{ GeV} \quad (29)$$

This leads to an estimate of the screening distance  $R_0$  from the relation  $V_{adj}(R_0) = 2M_{GL}$  to be  $R_0 \approx 1.4 \text{ Fm}$ , which is beyond the distance where Casimir scaling was measured in [1]. It is interesting to note that a similar estimate of gluelump mass for higher  $D$  leads to the decreasing  $R_0(D)$ , e.g. for  $D = 15s$  one gets  $R_0 = 0.7 \text{ Fm}$  which is not in contradiction with the data from [1].

## 6 Discussion

The Casimir scaling behaviour of the confining potential confirmed in [1] with the unprecedented precision leads to many important consequences, some of which have been discussed in the present paper. It is instructive to compare the pictures of the QCD vacuum, suggested in different models from the Casimir scaling point of view.

Abelian projection language being in wide use nowadays as one of the most adequate for the dual Meissner scenario of confinement encounters difficulties in explanation of the Casimir scaling. The reader is referred to the paper [19] where the question is discussed in details for the adjoint static charges. The observed adjoint string tension (at intermediate distances) arises from the interaction of diagonal abelian projected gluons with the part of the adjoint source doubly charged with respect to the Cartan subgroup. If one naively omits the corresponding Faddeev–Popov determinant it gives  $\sigma_{adj} = 4\sigma_{fund}$ . It is expected, that the loop expansion of the determinant produces terms, correcting the above behaviour to the Casimir scaling relation. Up to the authors’ knowledge, it has not yet been shown analytically, while there are numerical evidences from the lattice in favour of this possibility (see [19] and references therein). From physical point of view to reproduce Casimir scaling, which is genuine nonabelian feature, one needs to restore the original nonabelian gauge invariance broken by hand in the abelian projected method.

The now popular confining mechanism is the model of fat center vortices [20]. While the original center vortex picture cannot explain confinement of the adjoint charges, the introduction of the finite thickness of the vortex

makes it possible, to obtain approximate Casimir ratios for the string tensions [20]. However, with the high accuracy of the data [1] these ratios are excluded.

In the gauge-invariant formalism [4], the Casimir scaling has two important features. First, it is the direct consequence of the Gaussian dominance hypothesis (see review [2]) since gaussian correlator provides the exact Casimir scaling. On the other hand, it implies the cancellations of  $C_D^2$  – proportional terms and higher ones in the cluster expansion (2). Physically, it means the picture of the vacuum, made of relatively small colour dipoles with weak interactions between them. One can imagine two possible scenario. According to the first one, Casimir scaling is the consequence of Gaussian dominance. In this case any higher cumulant contributes to physical quantities much less than the gaussian one due to dynamical reasons. There is also the second possibility, when each higher term in the expansion (2) is not small, but their sum demonstrates strong cancellations of Casimir scaling violating terms. These pictures are in close correspondence to the stochastic versus coherent vacuum scenario [2]. This set of questions certainly deserves further study.

There are also several open questions of computational origin. Additional measurements are needed in order to clarify the validity of scaling for higher representations where the statistics is still rather poor. It would also be very interesting to establish the adjoint string breaking scale in  $SU(3)$  which could shed some light on the gluelumps physics.

Needless to say, that deeper theoretical understanding of the QCD vacuum structure is still required. The ability to incorporate such nontrivial feature as Casimir scaling is a necessary property of any reasonable confinement model.

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Figure 1: The Casimir scaling results for the adjoint potential, based on the data from [1]: axes –  $(V_8(R) - 2.25 \cdot V_3(R))$ ; solid line – leastsquare fit according to (8); dashed line – scaling violating contribution from the SV term (20). All quantities are given in the lattice units.

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